

CU-TP-684
 LUNFD6/(NFFL-7106) 1995
 hep-ph/9503494

Bose-Einstein Correlations for Expanding Finite Systems

T. Csörgő^{a,b} and B. Lörstad^b

^a *MTA KFKI RMKI,
 H-1525 Budapest 114, P.O.Box 49
 and*

*Department of Physics,
 Columbia University, New York, NY 10027*

^b *Department of Physics, University of Lund,
 Sölvegatan 14A, S-223 62 Lund, Sweden*

March 11, 1995

Abstract: There are *two length-scales* present simultaneously in all the principal directions for three-dimensionally expanding, finite systems. These are discussed in detail for the case of a longitudinally expanding system with a transverse flow and a transverse temperature profile. For systems with large geometrical sizes we find an m_t -*scaling* for the parameters of the Bose-Einstein correlation function, which may be valid in the whole transverse mass region for certain model parameters. In this limit, the Bose-Einstein correlations view only a small part of the source. The large geometrical sizes can be inferred from a simultaneous analysis of the invariant momentum distribution and the Bose-Einstein correlation function. A preliminary analysis of the NA44 data indicates that instead of a small fireball we are observing a big and expanding *snowball* in $S + Pb$ reactions at CERN SPS energies.

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T. Csörgő^{a,b} * and B. Lörstad^b

^a MTA KFKI RMKI, H-1525 Budapest 114, P.O.B. 49, Hungary;
Department of Physics, Columbia University, New York, USA

^b Department of Physics, University of Lund,
Sölvegatan 14A, S-223 62 Lund, Sweden

Bose-Einstein correlations and invariant momentum distributions are presented for expanding finite systems with applications to recent NA44 data.

1. Introduction

Bose-Einstein correlation functions (BECF-s) provide a unique tool for the analysis of the freeze-out geometry on the fermi scale. They are also subject to lots of non-ideal effects which make the analysis and interpretation of these data rather difficult.

There are *two length-scales* present simultaneously in all the principal directions of three dimensionally expanding, finite systems [1]. One of the length-scales is the geometrical size R_G , the other is generated by the freeze-out temperature distribution and the gradients of the flow. This second kind of radius is referred to either as the 'thermal radius' R_T [1–3] or the 'length of homogeneity' [4,5]. The thermal radius characterizes the size of the region in the fluid from which particles with similar momentum are emitted, while the geometrical radius is present due to the finite size of the expanding system.

2. Model specification

We model the emission function for high energy heavy ion reactions as

$$\begin{aligned}
 S(x, K) d^4x &= \frac{g}{(2\pi)^3} m_t \cosh(\eta - y) \exp\left(-\frac{K \cdot u(x)}{T(x)} + \frac{\mu(x)}{T(x)}\right) H(\tau) d\tau \tau_0 d\eta dr_x d\eta_\perp \\
 u(x) &\simeq \left(\cosh(\eta) \left(1 + b^2 \frac{r_x^2 + r_y^2}{2\tau_0^2}\right), b \frac{r_x}{\tau_0}, b \frac{r_y}{\tau_0}, \sinh(\eta) \left(1 + b^2 \frac{r_x^2 + r_y^2}{2\tau_0^2}\right) \right) \\
 T(x) &= \frac{T}{1 + a^2 \frac{r_x^2 + r_y^2}{2\tau_0^2}} \quad \text{and} \quad \frac{\mu(x)}{T(x)} = \frac{\mu_0}{T} - \frac{r_x^2 + r_y^2}{2R_G^2} - \frac{(\eta - y_0)^2}{2\Delta\eta^2}. \quad (3)
 \end{aligned}$$

Thus we include a finite duration, $H(\tau) \propto \exp(-(\tau - \tau_0)^2/(2\Delta\tau^2))$. The decrease of the temperature in the transverse direction is controlled by the parameter a , while the

*Supported in part by grants OTKA-T2973, F4019 and W015107, MAKKA 378/93 and Soros C5309/94

strength of the transverse flow is controlled by the parameter b . A non-relativistic limit for $a = 0$ is described in ref. [1]. One dimensionally expanding finite systems correspond to the $a = b = 0$ case [2], see this paper for the notation too. The first version of the three dimensionally expanding cylindrically symmetric finite model corresponds to the $a = 0$ and $b = 1$ case [3]. The integrals of the emission function are evaluated using the saddle-point method [4–6]. The saddle-point equations are solved in the LCMS [2], the longitudinally comoving system, for $\eta_s \ll 1$ and $r_{x,s} \ll \tau_0$. These approximations are warranted if $|y - y_0| \ll 1 + \Delta\eta^2 m_t / T$ and $\beta_t = p_t / m_t \ll (a^2 + b^2) / b$. The flow is non-relativistic at the saddle-point if $\beta_t \ll (a^2 + b^2) / b^2$. The radius parameters are evaluated here up to $\mathcal{O}(r_{x,s} / \tau_0) + \mathcal{O}(\eta_s)$, keeping only the leading order terms in the LCMS. However, when evaluating the invariant momentum distribution (IMD), sub-leading terms coming from the $\cosh(\eta - y)$ pre-factor are also summed up, since this factor influences the IMD in the lower m_t region where the data are very accurate.

2.1. Bose-Einstein correlations

The BECF is parameterized in the form of $C(Q_L, Q_{side}, Q_{out}) = 1 + \lambda \exp(-R_L^2 Q_L^2 - R_{side}^2 Q_{side}^2 - R_{out}^2 Q_{out}^2)$ where the intercept parameter λ and the radius parameters may depend on the rapidity and the transverse mass of the pair. We obtain [2,3]

$$R_{side}^2 = R_*^2, \quad R_{out}^2 = R_*^2 + \beta_t^2 \Delta\tau^2, \quad R_L^2 = \tau_0^2 \Delta\eta_*^2, \quad (4)$$

$$\frac{1}{R_*^2} = \frac{1}{R_T^2} + \frac{1}{R_G^2} \quad \text{and} \quad \frac{1}{\Delta\eta_*^2} = \frac{1}{\Delta\eta^2} + \frac{1}{\Delta\eta_T^2}, \quad (5)$$

i.e. the parameters of the BECF are dominated by the smaller of the geometrical and the thermal length-scales. The thermal length-scales R_T and $\Delta\eta_T$ are found to be

$$R_T^2 = \frac{\tau_0^2}{a^2 + b^2} \frac{T}{m_t} \quad \text{and} \quad \Delta\eta_T^2 = \frac{T}{m_t}. \quad (6)$$

These analytic expressions indicate that the BECF views only part of an expanding source. Even a complete measurement of the parameters of the BECF as a function of the mean momentum of the particles may be insufficient to determine uniquely the underlying space-time emission function due to this reason. If the finite source sizes are large compared to the thermal length-scales and if we also have $a^2 + b^2 \approx 1$, one obtains an m_t -scaling for the parameters of the BECF,

$$R_{side}^2 \simeq R_{out}^2 \simeq R_L^2 \simeq \tau_0^2 \frac{T}{m_t}, \quad \text{valid for } \beta_t \ll \frac{(a^2 + b^2)}{b^2} \simeq \frac{1}{b^2}. \quad (7)$$

Note that this relation is independent of the particle type and has been seen in NA44 data [7]. This m_t -scaling may be valid to arbitrarily large transverse masses with $\beta_t \approx 1$ if $b^2 \ll 1$. The finiteness of the expanding system reveals itself in an out-long cross term too [6] which is next to leading order in the LCMS, being smaller than $R_{out}^2 - R_{side}^2$, which measured difference is very small [7], alternatively explained by ref. [8].

2.2. Invariant momentum distributions

The IMD plays a *complementary role* to the measured Bose-Einstein correlation function [1–3]. Namely, the width of the rapidity distribution at a given m_t as well as T_* the effective

temperature at a mid-rapidity y_0 shall be dominated by the *longer* of the thermal and geometrical length-scales. Thus a *simultaneous analysis* of the Bose-Einstein correlation function and the IMD may reveal information both on the temperature and flow profiles and on the geometrical sizes. E.g. the following relations hold:

$$\Delta y(m_t)^2 = \Delta\eta^2 + \Delta\eta_T^2(m_t), \quad \text{and} \quad \frac{1}{T_*} = \frac{f}{T + T_G(m_t = m)} + \frac{1-f}{T}. \quad (8)$$

The geometrical contribution to the effective temperature is given by $T_G = T R_G^2/R_T^2$ and the fraction f is defined as $f = b^2/(a^2 + b^2)$, satisfying $0 \leq f \leq 1$. The saddle-point sits at $\eta_s = (y_0 - y)/(1 + \Delta\eta^2/\Delta\eta_T^2)$, $r_{x,s} = \beta_t b R_*^2/(\tau_0 \Delta\eta_T^2)$ and $r_{y,s} = 0$.

For the considered model, the invariant momentum distribution can be calculated as

$$\begin{aligned} \frac{d^2 n}{dy dm_t^2} = & \frac{g}{(2\pi)^3} \exp\left(\frac{\mu_0}{T}\right) m_t (2\pi \Delta\eta_*^2 \tau_0^2)^{1/2} (2\pi R_*^2) \cosh(\eta_s) \exp(+\Delta\eta_*^2/2) \times \\ & \times \exp\left(-\frac{(y - y_0)^2}{2(\Delta\eta^2 + \Delta\eta_T^2)}\right) \exp\left(-\frac{m_t}{T} \left(1 - f \frac{\beta_t^2}{2}\right)\right) \exp\left(-f \frac{m_t \beta_t^2}{2(T + T_G)}\right) \end{aligned} \quad (9)$$

This IMD has a rich structure: it features both a rapidity-independent and a rapidity-dependent low-pt enhancement as well as a high-pt enhancement or decrease.

Within this model, the *rapidity-independent low-pt enhancement* is a consequence of the transverse mass dependence of the effective volume, from which particles with a given momentum are emitted. This is described by the $(2\pi \Delta\eta_*^2 \tau_0^2)^{1/2} (2\pi R_*^2) \cosh(\eta_s) \exp(+\Delta\eta_*^2/2)$ factor, being proportional to $(T/m_t)^{(3/2)} \exp(+T/(2m_t))$.

The *rapidity-dependent low-pt enhancement*, which is a generic property of the longitudinally expanding finite systems [9], reveals itself in the rapidity-dependence of the effective temperature, defined as the slope of the exponential factors in the IMD in the low-pt limit at a given value of the rapidity. The leading order [9] result is

$$T_{eff}(y) = \frac{T_*}{1 + a(y - y_0)^2} \quad \text{with} \quad a = \frac{T T_*}{2m^2} \left(\Delta\eta^2 + \frac{T}{m} \right)^{-2}. \quad (10)$$

The *high-pt enhancement or decrease* refers to the change of the effective temperature at mid-rapidity with increasing m_t . The large transverse mass limit T_∞ shall be in general different from the effective temperature at low pt given by T_* since

$$T_\infty = \frac{2T}{2-f} \quad \text{and} \quad \frac{T_\infty}{T_*} = \frac{2}{2-f} \left(1 - f \frac{T_G(m)}{T + T_G(m)} \right). \quad (11)$$

Utilizing $T_G/T = R_G^2/R_T^2$, the high-pt enhancement or decrease turns out to be controlled by the ratio of the thermal radius $R_T(m_t = m)$ to the geometrical radius R_G . One obtains $T_\infty > T_*$ if $R_T^2(m) > R_G^2$ and similarly $T_\infty < T_*$ if $R_T^2(m) < R_G^2$. Since for large colliding nuclei R_G is expected to increase, the high-pt decrease in these reactions becomes a geometrical effect, a consequence of the large size.

3. Large Halo: Applications to NA44 data

The effects of long-lived resonances on both the IMD and the BECF can be taken into account analytically along the lines of [10], supposing that the halo is sufficiently large. NA44 data [7] for central $S + Pb$ reactions at CERN SPS with 200 AGeV show an approximately m_t independent intercept parameter: $\lambda_{\pi+} = 0.56 \pm 0.02$ and 0.55 ± 0.02 at $m_t = 150$ MeV and 450 MeV, respectively. This suggests that the halo contains $1 - \sqrt{\lambda} = 25 \pm 2$ % of all the pions. If $\lambda = const$ the equations simplify [10] as

$$\frac{d^2 n}{dy dm_t^2} = \frac{d^2 n_c}{dy dm_t^2} = \frac{d^2 n_h}{dy dm_t^2}, \quad \text{and} \quad C(\Delta k, K) = 1 + \lambda R_c(\Delta k, K). \quad (12)$$

In this case the only apparent effect of the halo is to reduce the intercept parameter of the measured BECF to $\lambda = const$ while the IMD and the $(\Delta k, K)$ dependence of the BECF is determined by the central part apparently *exclusively*. This central part is well accessible to the analytical models, presented e.g. in the previous sections. In general the halo may have more influence both on the IMD and the BECF.

In a preliminary analysis, we have fitted the NA44 preliminary IMD for pions and kaons together with the final NA44 data [7] for the m_t dependence of the BECF parameters for both pions and kaons. Fixing the parameters $y_0 = 3$, $a = 0$ and $b = 1$ and extrapolating the equations to the relativistic m_t domain we get a description of the IMD and the BECF for both pions and kaons at a $\chi^2/NDF = 2.2$. The preliminary analysis indicates large geometrical source sizes for the pions, $R_G(\pi) \approx 8$ fm, late freeze-out times of $\tau_0 \approx 8$ fm/c, a vanishing duration $\Delta\tau$ for both pions and kaons and finally surprisingly low freeze-out temperatures, $T \approx 80$ MeV. The kaons appear from a smaller region of $R_G(K) \approx 3.5$ fm.

4. Conclusions:

Instead of observing a small fireball we are observing a big and expanding *snowball* at CERN SPS $S + Pb$ reactions according to the above preliminary analysis of the partly preliminary NA44 data. The snowball may contain a hot core, to be investigated in a future data analysis. The preliminary results are very sensitive to the detailed structure of the m_t -dependence of the parameters of the Bose-Einstein correlation functions as well as to the details of the invariant momentum distribution of both pions and kaons.

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